Review Quiz 1

Instructions. You have 15 minutes to complete this review quiz. You may <u>not</u> use your calculator. You may not use any other materials. Submit your answers using the provided Google Form.

1. If the cross product of two nonzero vectors is $\langle 0, 0, 0\rangle$, what can we conclude about the vectors?

   (a) Nothing – not enough information.
   (b) They are orthogonal.
   (c) They are parallel.
   (d) They are unit vectors.
   (e) The vectors have the same magnitude.

   Recall: $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$, where θ is the angle between $\vec{a}$ and $\vec{b}$.

   If $\vec{a} \times \vec{b} = \langle 0, 0, 0\rangle \Rightarrow |\vec{a} \times \vec{b}| = 0 \Rightarrow \sin\theta = 0$

   $\Rightarrow \theta = 0 \Rightarrow \vec{a}$ and $\vec{b}$ must be parallel.

2. Which vector is orthogonal to $\langle 1, 3, 2\rangle$?

   (a) $\langle 1, 1, 1\rangle$
   (b) $\langle 0, 1, 0\rangle$
   (c) $\langle 1, -1, 1\rangle$
   (d) $\langle -1, 0, 1\rangle$
   (e) $\langle 2, 3, 1\rangle$

   Recall: $\vec{a}$ and $\vec{b}$ are orthogonal if $\vec{a} \cdot \vec{b} = 0$

   $\langle 1, 3, 2\rangle \cdot \langle 1, -1, 1\rangle = 1 - 3 + 2 = 0$

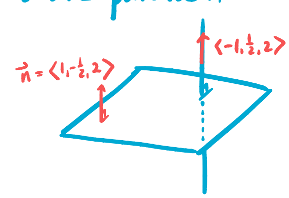
3. Which of these planes is perpendicular to the line $x = 2 - t,\ y = -2 + \frac{1}{2}t,\ z = 1 + 2t$?

   (a) $x - \frac{1}{2}y - 2z = 5$
   (b) $2x - 2y + z = 3$
   (c) $x - 2y - \frac{1}{2}z = 8$
   (d) $-\frac{1}{2}x + \frac{1}{2}y - z = 7$
   (e) $2x + z = 4$

   ↳ This line has direction vector $\langle -1, \frac{1}{2}, 2\rangle$

   This plane has normal vector $\langle 1, -\frac{1}{2}, -2\rangle$

   These 2 vectors are parallel.



4. The tangent vector to the curve $\vec{r}(t) = \langle 2t, \sin t, \cos t\rangle$ at $t = \pi$ is:

   (a) $\langle 2\pi, -\pi, 0\rangle$
   (b) $\langle 2, -1, 0\rangle$
   (c) $\langle 2, 0, 1\rangle$
   (d) $\langle 2\pi, 0, 1\rangle$
   (e) $\langle 2\pi, -1, 0\rangle$

   $\vec{r}\,'(t) = \langle 2, \cos t, -\sin t\rangle$

   $\Rightarrow \vec{r}\,'(\pi) = \langle 2, -1, 0\rangle$

5. Find the length of the curve $\vec{r}(t) = \langle \sin t, \cos t, t\sqrt{3}\rangle$ from $t = 0$ to $t = 10$.

   (a) $10 + 50\sqrt{t}$
   (b) $\cos(10) + \sin(10) + 10\sqrt{3}$
   (c) $10 + 10\sqrt{3}$
   (d) 10
   (e) 20

   $L = \int_0^{10} |\vec{r}\,'(t)|\, dt = \int_0^{10} 2\, dt = 20$

   $\vec{r}\,'(t) = \langle \cos t, -\sin t, \sqrt{3}\rangle$

   $\Rightarrow |\vec{r}\,'(t)| = \sqrt{\cos^2 t + \sin^2 t + 3}$

   $= \sqrt{1 + 3} = 2$